Third Order Magic Square ([anqisoft@gmail.com](mailto:anqisoft@gmail.com))

To enumerate all the answers: There are 9 numbers now, with equal differences increasing (such as 1-9). They are placed in the grid of the Nine Palaces, with eight horizontal, vertical and diagonal lines. The sums of the three numbers are all the same.

Solution: According to the English abbreviation, mark each box as follows:

|  |  |  |
| --- | --- | --- |
| NW | N | NE |
| W | C | E |
| SW | S | SE |

Step 1: First find the sum of nine numbers

NW+N+NE+W+C+E+SW+S+SE

=1+2+3+4+5+6+7+8+9

=45

Step 2: Find the phantom sum (any line of eight lines, the sum of three numbers)

45÷3=15

Step 3: Find the phantom heart again (the number that hits the right square)

Sum of four lines through the phantom: 15×4=60

Expand four lines:

(N+C+S) + (W+C+E) + (NW+C+SE) + (NE+C+SW)

= (NW+N+NE+W+C+E+SW+S+SE) + C × 3

= 45 + C × 3

∴ C = (60-45) ÷3=15÷3=5

It can be seen from this: N+S=W+E=NW+SE=NE+SW=15-5=10

Step 4: Prove the position of 1 (sides, corners)

If 1 can be set as an angle, and NW, NE, SW, and SE are equivalent, choose NW for analysis.

∵ NW+SE=10

∴ SE = 9

∴ The remaining 6 numbers: 2, 3, 4, 6, 7, 8

∵ N+NE=W+SW=15-1=14, the above six numbers can only be combined to form the group of 6 and 8

∴ 1 cannot set corners, only edges

Step 5: Place 1 on the edge and enumerate the results. Set 1 to N as an example,

|  |  |  |
| --- | --- | --- |
| NW | 1 | NE |
| W | 5 | E |
| SW | 9 | SE |

NW is 6 or 8, and the corresponding NE is 8 and 6. Because NW+SE=10, SE is 4 and 2. Because NE+SW=10, SW is 2 and 4. Because NW+W+SW=15, W is 7 and 3. Since W+E=10, E is 3 and 7. Therefore, we get this solution:

|  |  |  |
| --- | --- | --- |
| 6 8 | 1 | 8 6 |
| 7 3 | 5 | 3 7 |
| 2 4 | 9 | 4 2 |

and extend it to three sides:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 6 8 | 1 | 8 6 |  | 2 4 | 7 3 | 6 8 |  | 2 4 | 9 | 4 2 |  | 6 8 | 7 3 | 2 4 |
| 7 3 | 5 | 3 7 | 9 | 5 | 1 | 7 3 | 5 | 3 7 |  | 1 | 5 | 9 |
| 2 4 | 9 | 4 2 | 4 2 | 3 7 | 8 6 | 6 8 | 1 | 8 6 |  | 8 6 | 3 7 | 4 2 |

三阶幻方 （[anqisoft@gmail.com](mailto:anqisoft@gmail.com)）

穷举所有答案：今有9数，等差递增（如1-9），令入九宫之格，横纵对角八线，三数之和皆同。

解：依于英文缩写，标记各格如下：

|  |  |  |
| --- | --- | --- |
| NW | N | NE |
| W | C | E |
| SW | S | SE |

步1：先求九数之和

NW+N+NE+W+C+E+SW+S+SE

=1+2+3+4+5+6+7+8+9

=45

步2：次求幻和（八线任意一线，三数之和）

45÷3=15

步3：再求幻心（正中一格之数）

过幻心四线和：15×4=60

展开四线：

(N+C+S) + (W+C+E) + (NW+C+SE) + (NE+C+SW)

= (NW+N+NE+W+C+E+SW+S+SE) + C × 3

= 45 + C × 3

∴ C = (60-45) ÷3=15÷3=5

由是可知：N+S=W+E=NW+SE=NE+SW=15-5=10

步4：求1之位（边、角）

若1可置角，NW、NE、SW、SE对等，择NW析之

∵ NW+SE=10

∴ SE = 9

∴ 余下6数：2、3、4、6、7、8

∵ N+NE=W+SW=15-1=14，此六数仅得6、8这组

∴ 1不可置角，只可置边

步5：置1于边，穷举结果。置1于N为例，

|  |  |  |
| --- | --- | --- |
| NW | 1 | NE |
| W | 5 | E |
| SW | 9 | SE |

NW取6或8，相应NE为8与6，因NW+SE=10而得SE为4与2，因NE+SW=10而得SW为2与4，因NW+W+SW=15而得W为7与3，因W+E=10而得E为3与7。故得此解，

|  |  |  |
| --- | --- | --- |
| 68 | 1 | 86 |
| 73 | 5 | 37 |
| 24 | 9 | 42 |

扩至三边：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 68 | 1 | 86 |  | 24 | 73 | 68 |  | 24 | 9 | 42 |  | 68 | 73 | 24 |
| 73 | 5 | 37 | 9 | 5 | 1 | 73 | 5 | 37 |  | 1 | 5 | 9 |
| 24 | 9 | 42 | 42 | 37 | 86 | 68 | 1 | 86 |  | 86 | 37 | 42 |

三階幻方 （[anqisoft@gmail.com](mailto:anqisoft@gmail.com)）

窮舉所有答案：今有9數，等差遞增（如1-9），令入九宮之格，橫縱對角八線，三數之和皆同。

解：依于英文縮寫，標記各格如下：

|  |  |  |
| --- | --- | --- |
| NW | N | NE |
| W | C | E |
| SW | S | SE |

步1：先求九數之和

NW+N+NE+W+C+E+SW+S+SE

=1+2+3+4+5+6+7+8+9

=45

步2：次求幻和（八線任意一線，三數之和）

45÷3=15

步3：再求幻心（正中一格之數）

過幻心四線和：15×4=60

展開四線：

(N+C+S) + (W+C+E) + (NW+C+SE) + (NE+C+SW)

= (NW+N+NE+W+C+E+SW+S+SE) + C × 3

= 45 + C × 3

∴ C = (60-45) ÷3=15÷3=5

由是可知：N+S=W+E=NW+SE=NE+SW=15-5=10

步4：求1之位（邊、角）

若1可置角，NW、NE、SW、SE對等，擇NW析之

∵ NW+SE=10

∴ SE = 9

∴ 餘下6數：2、3、4、6、7、8

∵ N+NE=W+SW=15-1=14，此六數僅得6、8這組

∴ 1不可置角，只可置邊

步5：置1于邊，窮舉結果。置1於N為例，

|  |  |  |
| --- | --- | --- |
| NW | 1 | NE |
| W | 5 | E |
| SW | 9 | SE |

NW取6或8，相應NE為8與6，因NW+SE=10而得SE為4與2，因NE+SW=10而得SW為2與4，因NW+W+SW=15而得W為7與3，因W+E=10而得E為3與7。故得此解，

|  |  |  |
| --- | --- | --- |
| 68 | 1 | 86 |
| 73 | 5 | 37 |
| 24 | 9 | 42 |

擴至三邊：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 68 | 1 | 86 |  | 24 | 73 | 68 |  | 24 | 9 | 42 |  | 68 | 73 | 24 |
| 73 | 5 | 37 | 9 | 5 | 1 | 73 | 5 | 37 |  | 1 | 5 | 9 |
| 24 | 9 | 42 | 42 | 37 | 86 | 68 | 1 | 86 |  | 86 | 37 | 42 |